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STRESS ANALYSIS OF LAMINATED
COMPOSITES WITH PREFORMED HOLES

PART ONE - SINGLE PLIES WITH EMBEDDED
MATERIAL DISCONTINUITIES

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SEPTEMBER 1992

FINAL REPORT FOR 05/26/92-09/30/92

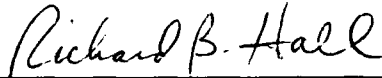APPROVED FOR PUBLIC RELEASE; DISTRIBUTION IS UNLIMITED.

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This technical report has been reviewed and is approved for publication.

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Stress Analysis of Laminated Composites with Preformed
Holes.   Part 1:   Single Plies with Material Disconti-   C: F33615-89-C-5609
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Work performed by AdTech Systems Research, Inc., under subcontract to UTC.

An Analytical and experimental study was conducted to develop a computational model
adequate for stress analyses of AS4/3501-6 composite plates containing preformed
1/4" diameter holes, which were found in previous and parallel experimental studies
to possess superior open-hole tension and compresssion strengths relative to
composites with drilled holes of equal diameter. The preformed-hole composites
contain embedded material discontinuities resulting from the interpenetration of
adjacent composite layers in the actual, laminated systems. The effort reported
addresses the initial development of the model for single layers containing the
material discontinuities present in laminates. The analytical method is suitable
for adaptation to problems involving smart meterials, electronic packaging, micro-
electromechanical systems and joining/fastening.
The proposed computational model is based on spline approximations of the displace-
ment components. Solutions were obtained relative to curvilinear coordinates that
mapped the preformed hole region onto a rectangular region and the embedded material
boundaries into coordinate lines. Functions capable of representing point discon-
tinuities in the strain field were used to satisfy the equilibrium conditions at the

Composites, holes, embedded material, variational method, numer-   27
ical analysis, spline functions, smart materials, electronic
packaging, MEMs, joining, fastening
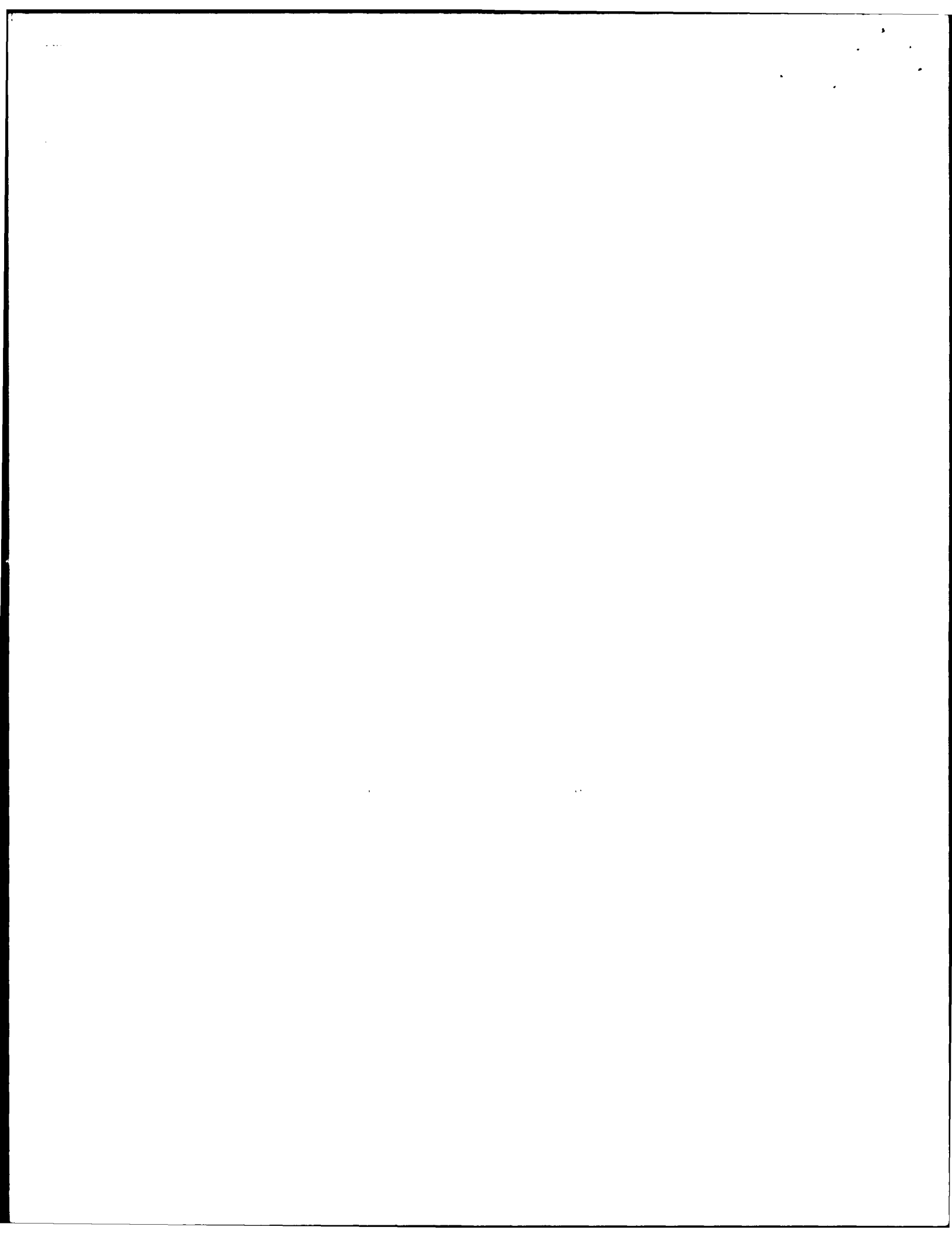
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interfaces associated with the embedded material. The results agree
qualitatively with the substantial, experimentally observed increases of
the tension and compression strengths of plates with preformed holes
compared to those with drilled holes.

ABSTRACT

Stress analysis in the vicinity of the preformed hole with the fibers smoothly modeled around the hole was performed. At this stage the stress analysis in a single ply was conducted. Spline approximation of displacements was employed. The solution was obtained in curvilinear coordinates which mapped the preformed hole region onto a simple rectangular region. Two predominant fiber orientations were considered: 0° and 90° AS4/3501-6 plates, both containing 1/4" diameter preformed hole and subjected to unidirectional tension in 0° direction. For the 0° ply, calculations were made assuming that the matrix-rich triangular-shape gaps left by the fibers surrounding the hole are filled with 90° or 45° fibers. And in the case of 90° plate, it was assumed that the gaps are filled by 0° or 45° fibers. The stresses in fiber direction and in the direction perpendicular to fibers were compared to those at the edge of the drilled hole. At least 40% stress reduction in the 0° ply case was observed for the stress in fiber direction at the edge of the preformed hole compared to the same stress at the drilled hole edge. The work was supported by AFOSR task #2303/DW , "Biotechnology for Aerospace Material Requirements." The project engineer was Dr. Richard Hall, WL/MLBM, Wright-Patterson AFB, OH 45433-6533. The contract monitor was Dr. Douglas S. Dudis, WL/MLBP, WPAFB.

INTRODUCTION

The application of biological ideas to improve the performance of composite materials has attracted researchers for a long time. Several papers were devoted recently to the strength analysis of the composites containing preformed holes with continuous fibers. In the paper [1] a stream line function is utilized to describe the fiber configuration around the moulded hole. A glass/polyester woven roving composite was considered. Unidirectional lamina with moulded hole was analyzed. Unfortunately the calculated stresses were not shown in the principal material direction and didn't allow any conclusions to be made upon the strength of the composites. A simple equivalent radius model for

1

estimation of the compression strength of the woven composite with the moulded hole was proposed in [2]. Point and average stress criteria were applied for strength prediction. To obtain the stress concentration factor at the moulded hole edge, "Equivalent Radius Model" was applied. For the moulded hole of a given radius, an equivalent radius of a drilled hole was obtained so that the compression strength for these two cases was equal. The composites with moulded holes were found to have higher compression strength, therefore the diameter of the equivalent drilled hole is smaller than the diameter of the actual moulded hole. After the equivalent diameter is found, the panel with the moulded hole is treated as a panel with a drilled hole but with a smaller hole diameter. An interesting problem is approached in [3]: Can the tensile capacity and/or the buckling capacities of a plate with a central hole be improved by using the curvilinear fiber format? What mechanisms are responsible for this improvement? A finite element approach was used. In every element a different direction of principle material axis was considered. The results of stress calculations were compared with standard quasi-isotropic design. It was shown that the curvilinear fiber format has the potential of improving the load carrying capacity of composites.

A common disadvantage of the models considered above is that they are not based on considering the real , multidirectional type of reinforcement which occurs around the preformed holes. Only the analysis based on a thorough experimental investigation of the structure of the composite with the preformed hole may allow to understand the mechanism of the increased or reduced strength of these composites depending upon the applied loading. Only in the experimental paper [4] was the multidirectional reinforcement discussed which was provided by the expansion of adjacent plies into voids, or matrix-rich regions formed by the fibers surrounding the hole. The additional structural investigation and photographs provided by WL/MLBM made possible the development of the model

proposed in the present report and used for the stress analysis in single ply composites with moulded in holes.

PROBLEM FORMULATION

Different plies of the de-plied 16-ply AS4/3501-6 [90/45/-45/0]$_{2s}$ specimen with a 1/4" preformed hole are shown schematically in Figure 1. Every ply can be divided into two regions. Region #1 is the region with approximately constant fiber volume fracture and fiber orientation except in the vicinity of the hole. According to [4] a 4% increase of fiber volume fracture in the local preformed hole region was found. The continuous fibers surrounding the hole region indicate a significant local change of fiber orientation. However, according to the photographs provided by WL/MLBM, at distances more than 0.1 - 0.15' inside region #1 from the gap boundary the fiber orientation is constant for the entire ply. The region #2 consists of two triangular-shape regions which do not contain the fibers from region #1. A sectioning study of the laminated specimen, performed by WL/MLBM, showed that these regions do not remain pure matrix regions in the laminate. They are filled up with the fibers of adjacent plies in the process of cure. It results in a multidirectional in-plane type of reinforcement around the hole which is responsible for the increase of the preformed hole tensile strength as compared to the drilled hole.

In the present report the stress analysis in single plies with preformed holes shown in Figure 1 was accomplished. To perform a satisfactory laminate analysis, a significant development of the computational procedure was needed and went beyond the time limitations defined by this contract.

METHOD OF SOLUTION

Displacement spline approximation in combination with curvilinear coordinate transformation was utilized to solve the problem. Consider an orthotropic rectangular plate

of length - L, width - A and thickness - H containing a circular open hole (Figure 2a). The gap boundary is formed by straight lines emanating from line $y = y_c$ at a given distance D from the center of the hole and ending at the circular hole edge tangential to it. Displacement boundary conditions are applied at the edges $x = 0, L$ so that

$$- u_x (0,y,z) = u_x (L,y,z) = \Delta.$$

$\Delta > 0$ is the given displacement of the edges generating a tensile uniaxial stress σ_0

$$\sigma_0 = \frac{1}{H \cdot A} \int_0^H \int_0^A \sigma_{xx} (0,y,z) \, dydz \tag{1}$$

The following curvilinear transformation of the x and y coordinates is utilized:

$$x = x_c + \sum_{i=1}^{5} F_i(r) \, X_i(\phi),$$

$$y = y_c + \sum_{i=1}^{5} F_i (r) \, Y_i(\phi) \tag{2}$$

$$-1 \leq r \leq 1, \, 0 \leq \phi \leq 2\pi$$

The curvilinear coordinates r and ϕ are generalized coordinates which transform the plate with the open hole into a rectangular region [- 1, 1] x [0, 2π]. It is very important that the gap boundary in these new coordinates is simply a coordinate line r=0. The open hole boundary is the coordinate line r=-1, and finally the coordinate line r=1 is the rectangular boundary of the plate.

The functions F_i (r) are defined by the following simple formulas:

4

$$F_1(r) = \begin{cases} r^2, -1 \leq r < 0 \\ 0, 1 \geq r \geq 0 \end{cases}$$

$$F_2(r) = \begin{cases} -2r(r+1), -1 \leq r < 0 \\ 0, 1 \geq r \geq 0 \end{cases}$$

$$F_3(r) = \begin{cases} (r+1)^2, -1 \leq r < 0 \\ (r-1)^2, 1 \geq r \geq 0 \end{cases} \tag{3}$$

$$F_4(r) = \begin{cases} 0, -1 \leq r < 0 \\ 2r(1-r), ' \geq r \geq 0 \end{cases}$$

$$F_5(r) = \begin{cases} 0, -1 \leq r < 0 \\ r^2, 1 \geq r \geq 0 \end{cases}$$

Graphics of these functions are shown on Figure 2b. The functions $X_i(\phi)$ and $Y_i(\phi)$ are as follows:

$$X_1(\phi) = d \cos(\phi - \phi_0)$$

$$Y_1(\phi) = d \sin(\phi - \phi_0)$$

The functions $X_3(\phi)$ and $Y_3(\phi)$ as well as $X_5(\phi)$, $Y_5(\phi)$ are expressed in terms of cubic splines, providing that they will describe the gap boundary and the rectangular boundary of the plate. These functions are shown in Figures 3a and 3b. The functions X_2, Y_2 and X_4, Y_4 are not independent and are defined as

$$X_2(\phi) = \frac{1}{2}(X_1 + X_3)$$

$$Y_2(\phi) = \frac{1}{2}(Y_1 + Y_3)$$

$$X_4(\phi) = \frac{3}{2}X_3 - \frac{1}{2}X_1 \tag{4}$$

$$Y_4(\phi) = \frac{3}{2}Y_3 - \frac{1}{2}Y_1.$$

One can obtain that the relations (4) provide that the derivatives

$$\frac{\partial x}{\partial r}, \frac{\partial y}{\partial r}$$

are continuous for any r and ϕ values, even though $\dfrac{\partial F_3}{\partial r}$ is discontinuous at $r = 0$. The coordinate lines of this transformation are shown in Figures 4a and 4b for the $0°$ ply and $90°$ ply with moulded holes.

STIFFNESS PROPERTIES

Let us consider the unidirectional ply material constants given in the matrix form $C_{(123)}$. The ply properties in a rotated coordinate system xyz can be obtained according to the formula:

$$C_{(xyz)} = T_\varepsilon^*(\alpha) \, C_{(123)} \, T_\varepsilon(\alpha)$$

$T_\varepsilon(\alpha)$ is the strain vector rotation matrix and α is the angle between the x - axis and fiber direction. This matrix is given in reference [5]. The star denotes the transpose of the matrix.

In order to describe the stiffness change around the hole region due to the local change of fiber orientation, the following assumptions were made. The local increase of the fiber volume fraction by 4% in the vicinity of the hole was neglected. The change of the fiber orientation was described as

$$\sigma(r,\phi) = \left\{ \begin{array}{l} \theta \text{ - ply orientation, } r \geq 0.2 \\ \theta\dfrac{r}{0.2} + \beta(\phi)\dfrac{0.2\text{-}r}{0.2}, \; 0 \geq r > 0.2 \end{array} \right\} \tag{5}$$

In formula (5), $\alpha(r, \phi)$ is the angle between the principal material axis direction and the x - axis at the point (r, ϕ), θ is the ply orientation. According to (5), α is a constant value for $r > 0.2$. For $r < 0.2$ the angle α is smoothly changing from θ to $\beta(\phi)$ at the gap edge. $\beta(\phi)$ is the declination of the gap boundary to the x - axis for a given value of ϕ. It can be obtained as

$$\beta(\phi) = \text{arctg}\left(\frac{\dfrac{dX_3}{d\phi}}{\dfrac{dY_3}{d\phi}}\right).$$

The coordinate line $r = 0.2$ is shown in Figure 3. The contour defined by $r = 0.2$ is approximately at a distance 0.1-0.15" from the edge of the gap. Thus for $r > 0.2$ the fiber orientation is constant and equal to ply orientation θ.

Considering the region inside the gap - $1 \leq r \leq 0$, we assume that it is filled with the same unidirectional AS4/3501 but having the fibers oriented in the direction of the adjacent plies in the laminate, i.e., we will consider the following cases:

for $\theta = 0°$, $\alpha = 90°$ or $45°$ if $- 1 \leq r < 0$

for $\theta = 90°$, $\alpha = 45°$ or $0°$ if $- 1 \leq r < 0$

DISPLACEMENT SPLINE APPROXIMATION

In order to solve the complicated problem formulated above, we are using the displacement spline approximation approach [6]. It is important to emphasize that by introducing the curvilinear transformation (2) we achieved two advantages: first, we transformed the complex boundaries of the plate with the gap into simple coordinate lines $r = - 1, 0, 1$; and secondly we described the change of fiber orientation near the gap using the simple relation (5). However we anticipate great difficulties by trying to describe the orthotropic stiffness properties in the curvilinear coordinates. Moreover we are not interested even in introducing the displacements in the curvilinear coordinates because these coordinates are just general curvilinear coordinates without any special importance from the mechanical point of view. Therefore the unknowns of this problem will still be the displacements in oxyz coordinates: $u_x (x, y, z)$, $u_y (x, y, z)$ and $u_z (x, y, z)$. We are going to write the strain energy expression in xyz coordinates. Then we will replace the integration variables x and y by r and ϕ. Let us introduce the following vectors:

7

$$\vec{\sigma}_{(xyz)} = (\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{yz}, \sigma_{xz}, \sigma_{yx})$$

$$\vec{\varepsilon}_{(xyz)} = (\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \varepsilon_{yz}, \varepsilon_{xz}, \varepsilon_{yx})$$

(6)

The standard linear relationship between displacements and small strains is utilized. Hook's law can be written in the oxyz coordinates as

$$\vec{\sigma}_{(xyz)} = C_{(xyz)} \, \vec{\varepsilon}_{(xyz)} \, .$$

(7)

The expression for the strain energy is as follows:

$$E = \int_0^H dz \int \int_{\Lambda/\Omega} \vec{\varepsilon}^* \, C_{(xyz)} \, \vec{\varepsilon} \, dxdy,$$

where Λ/Ω denotes the rectangular area with the circular hole. The stiffness matrix $C_{(xyz)}$ is changing from point to point near the gap in accordance with (5). The strain components in this formula must be expressed through the displacements. Now we replace the integration variables x and y by r and ϕ so that

$$E = \int_0^H dz \int_{-1}^1 dr \int_0^{2\pi} \vec{\varepsilon}^* \, T_\varepsilon^*(\alpha(r,\phi)) \, C_{(123)} \, T_\varepsilon(\alpha(r,\phi)) \cdot \vec{\varepsilon} \det(J) \, d\phi$$

(8)

To complete this formulation we need to express $\vec{\varepsilon}$ components through u_x, u_y, u_z in r, ϕ, z coordinates. The Jacobian can be represented as:

$$J = \begin{pmatrix} \dfrac{\partial x}{\partial r} & \dfrac{\partial y}{\partial r} \\ \dfrac{\partial x}{\partial \phi} & \dfrac{\partial y}{\partial \phi} \end{pmatrix}$$

In order to express the $\vec{\varepsilon}$ components through u_x (r, ϕ, z), u_y (r, ϕ, z) and

8

u_z (r, ϕ, z), the following relations can be used:

$$\begin{pmatrix} \dfrac{\partial u_x}{\partial x} \\[2mm] \dfrac{\partial u_x}{\partial y} \end{pmatrix} = J^{-1} \begin{pmatrix} \dfrac{\partial u_x}{\partial r} \\[2mm] \dfrac{\partial u_x}{\partial \phi} \end{pmatrix} ; \begin{pmatrix} \dfrac{\partial u_y}{\partial x} \\[2mm] \dfrac{\partial u_y}{\partial y} \end{pmatrix} = J^{-1} \begin{pmatrix} \dfrac{\partial u_y}{\partial r} \\[2mm] \dfrac{\partial u_y}{\partial \phi} \end{pmatrix} ; \begin{pmatrix} \dfrac{\partial u_z}{\partial x} \\[2mm] \dfrac{\partial u_z}{\partial y} \end{pmatrix} = J^{-1} \begin{pmatrix} \dfrac{\partial u_z}{\partial r} \\[2mm] \dfrac{\partial u_z}{\partial \phi} \end{pmatrix} . \tag{9}$$

Using these relations we can write (8) in terms of displacements u_x, u_y, u_z in r, ϕ, z coordinates. Spline approximation of displacements upon r, ϕ and z is used. The spline approximation nodes are introduced in r, ϕ and z directions so that

$$- 1 = r_0 < r_1 < \ldots < r_K = 0 < r_{K+1} < \ldots < r_M = 1,$$

$$0 = \phi_0 < \phi_1 < \ldots < \phi_\rho = 2\pi$$

$$0 = z_0 < z_1 < \ldots < z_N = H.$$

Cubic spline functions shown in Figure 5a, 5b and 5c are introduced in r, ϕ and z directions respectively. Note that the set of functions in r direction incorporates the discontinuity of slope at r = 0, which is the boundary between two materials. In the ϕ direction the periodicity conditions are required at $\phi = 0$ and $\phi = 2\pi$. The displacement spline approximation can be written as

$$u_x (r,\phi,z) = \sum_{i=0}^{N+2} \sum_{j=0}^{M+4} \sum_{k=1}^{P} u_{ijk} Z_i(z) R_j(r) \Phi_k(\phi)$$

$$u_y (r,\phi,z) = \sum_{i=0}^{N+2} \sum_{j=0}^{M+4} \sum_{k=1}^{P} v_{ijk} Z_i(z) R_j(r) \Phi_k(\phi) \tag{10}$$

$$u_z (r,\phi,z) = \sum_{i=0}^{N+2} \sum_{j=0}^{M+4} \sum_{k=1}^{P} w_{ijk} Z_i(z) R_j(r) \Phi_k(\phi)$$

The equations for calculating the unknown displacement approximation coefficients are derived by substituting (10) into (8) and requiring the first variation of E to be zero:

$$\delta (E) = 0.$$

After displacement spline approximation coefficients are calculated, the displacement values in any point can be obtained from (10). Then the strains can be calculated using (9) and the stresses can be obtained using Hook's law (7). This completes the solution procedure.

NUMERICAL RESULTS

The AS4/3501-6 unidirectional ply properties utilized in calculations are given below:

$E_1 = 20$ Msi $X = 210$ ksi

$E_2 = E_3 = 1.7$ Msi $X_d = 210$ ksi

$G_{12} = G_{13} = 0.9$ Msi $Y = 7.5$ ksi

$G_{23} = 0.48$ Msi $Y_d = 29$ ksi

$\nu_{12} = \nu_{13} = 0.32$ $S = 13.5$ ksi

$\nu_{23} = 0.55$

The geometric parameters shown in Figure 2.a were obtained by measuring the specimens provided by WL/MLBM and found to be equal to

$$d = 1/4", D = 0.5", L = 5", A = 2.5".$$

Since we are considering a single ply and focusing on the in-plane stress component calculation, the thickness is not an important parameter. $H = 0.04"$ was used . The displacement applied at the edges $x = 0,L$ was equal to $\Delta = 0.2 \cdot 10^{-3}$. It resulted in the unidirectional load $\sigma_0 = 1595$ psi in $0°$ ply. Same displacement was applied to the $90°$ ply and the axial stress was $\sigma_0 = 139$ psi. Since ultimately we are interested in stress analysis in the laminate we will normalize all stresses in both plies to the value $\sigma_0 = 1595$ psi. In Figure 6 the stresses at the gap boundary in region #1 are shown for $0°$ ply with a moulded hole. Two cases are considered: when the gap is filled by the unidirectional composite with fibers in $90°$ direction and in $45°$ direction. The stresses in the direction of fibers at the gap boundary are compared with those at the edge of the drilled hole in $0°$ ply. The

10

angle is counted counterclockwise from the x-axis direction. Note that the stresses for the moulded hole are not at the same positions as in the drilled hole except the interval from 78° to 112° where the gap edge coincides with the hole edge. However even there the fiber orientation in the case of the moulded hole is continuously changing around the hole. Approximately 40% stress reduction occurs in the case of the moulded hole. The importance of this result is provided by the fact that the examined stress component controls the tensile strength of the 0° ply with the hole. The reduction of the stress concentration indicates that one reason for the increased tensile strength of the laminates with moulded holes vs. drilled hole is more efficient stress redistribution within the 0° plies due to multidirectional in-plane reinforcement. The stresses in the direction perpendicular to fibers at the edge of the hole are shown in Figure 7. At this time we are looking at normal stresses in the direction perpendicular to fibers at the gap boundary , versus same stress component at the edge of the drilled hole, and thereby we black out the region where the gap edge coincides with the hole edge. Higher stresses are indicated in case of the moulded hole than for the drilled hole. Practical importance of this conclusion is however questionable because it may be a mathematical effect due to singularity at the point where the gap edge and the hole edge meet , forming a sharp edge in region #2. The results for 90° ply are presented in Figures 8 and 9. In this case the failure is obviously controlled by the stresses in region #1 in the direction perpendicular to fiber which is the loading direction. As well as for the 0° ply a significant reduction of the critical stress component at the edge of the moulded hole is evident as compared to stress at the edge of the drilled hole, Figure 8. In Figure 9 the stress in fiber direction in region #2 is shown at the hole edge. The highest stress is indicated for the moulded hole with the gap filled with 0° fibers. However if we imagine the 0° and 90° ply with in a laminate and compare the maximal σ_{11}/σ_0 in Figures 6 and 9 we can see that the stress in the 90° ply with moulded hole is much smaller than in 0° ply and is not a factor of strength reduction.

CONCLUSIONS

1. A method based on displacement spline approximation and curvilinear coordinate transformation was proposed and developed for stress analysis in single orthotropic ply with moulded hole. The region of the gap or void formed by fibers surrounding the hole can be filled with the unidirectional composite with arbitrary fiber orientation.

2. Stress analysis in two unidirectional plies with fiber orientation of $0°$ and $90°$ with moulded holes under uniaxial tension in $0°$ direction was performed. The results were compared to the stresses at the drilled hole edge calculated using a previously developed program. In the $0°$ ply approximately 40% of the reduction of the failure controlling stress component (stress in fiber direction) was obtained as compared to the drilled hole. For $90°$ ply a 15% reduction of the failure controlling stress component (stress in direction perpendicular to fibers) was observed.

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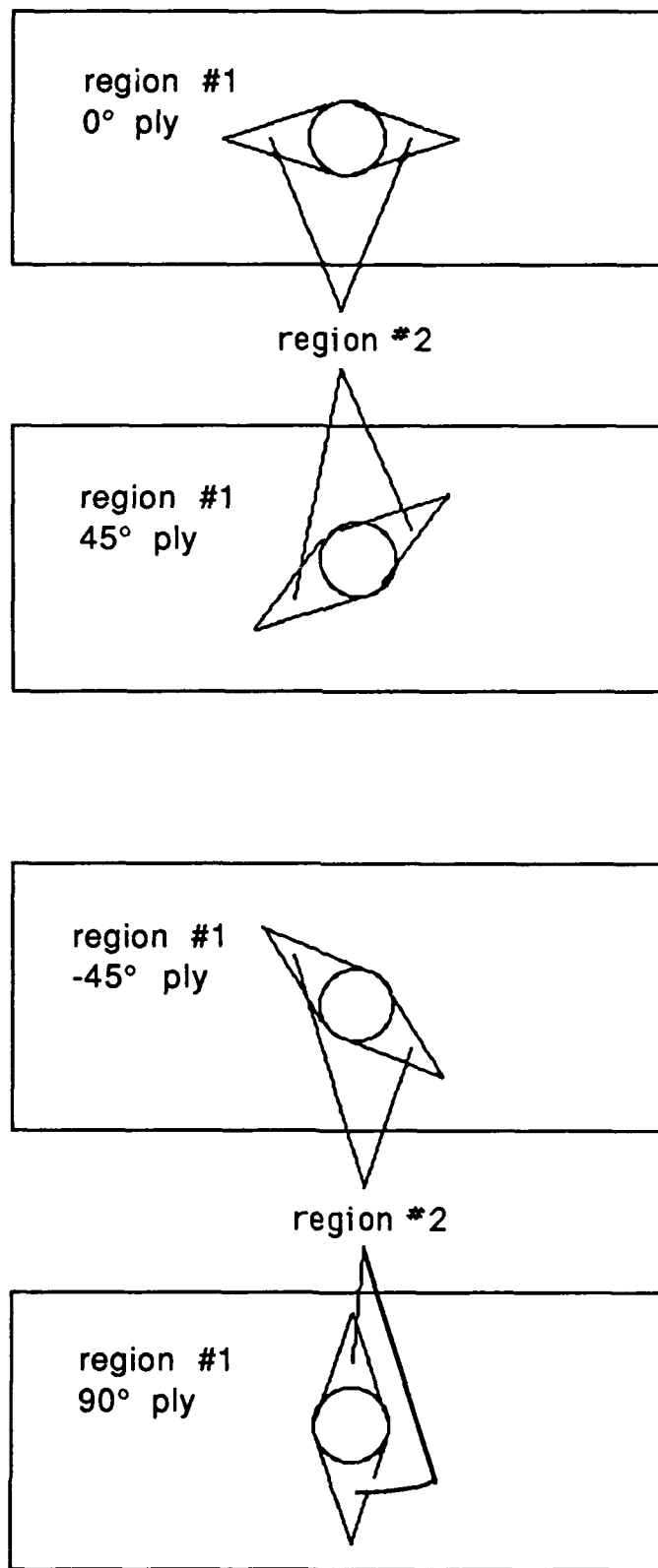
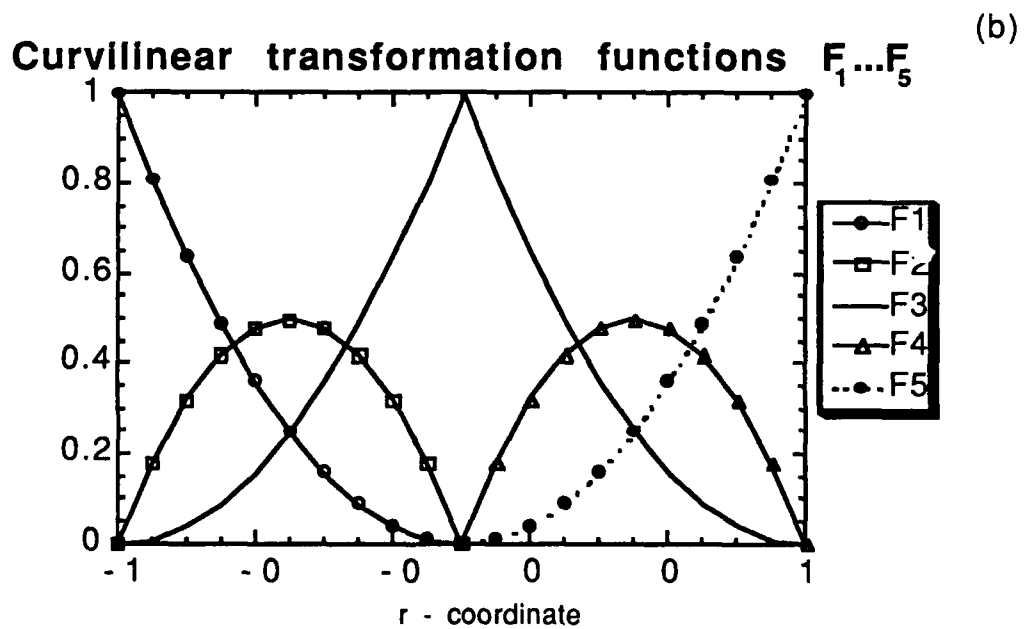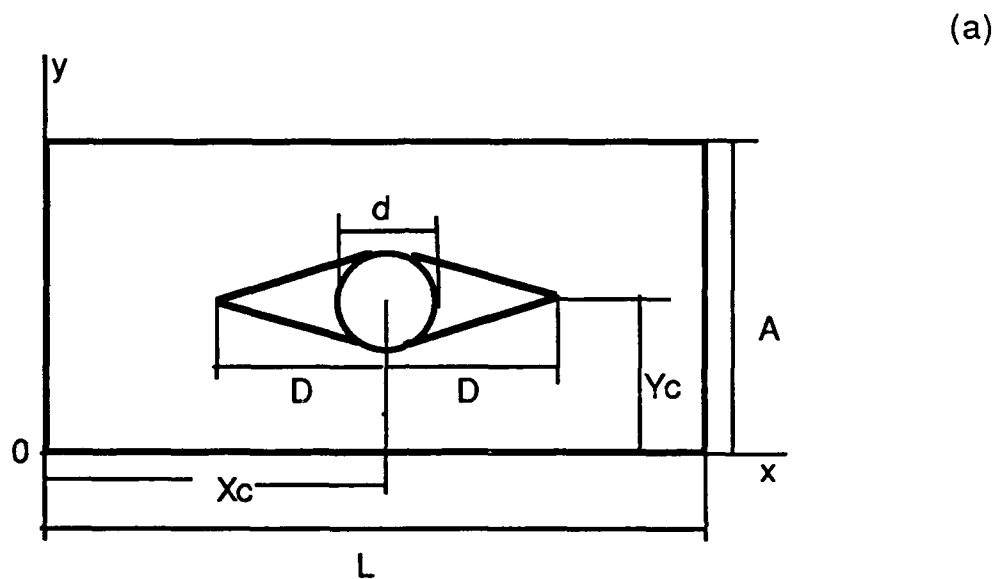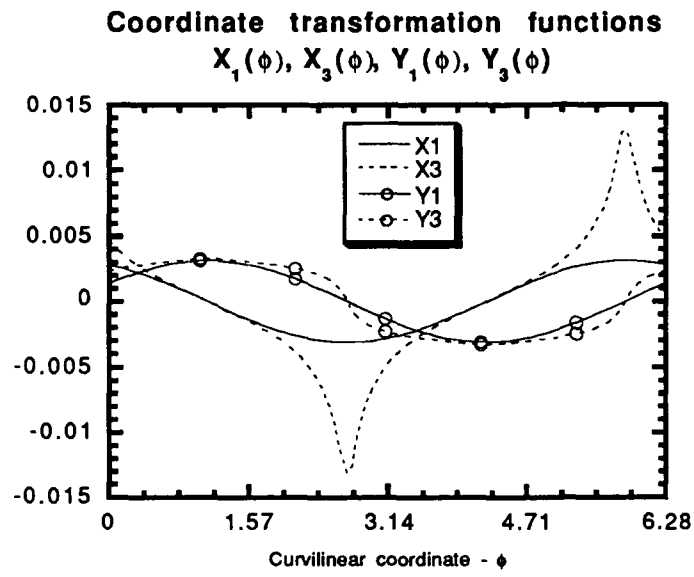Figure 1. Several plies of the de-plied 16-ply $[90/\pm45/0]_{2s}$ AS4/3501-6 laminate

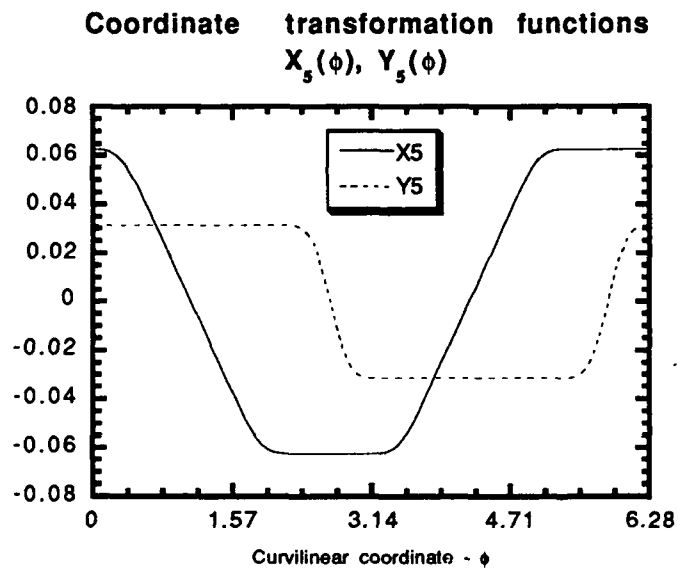Figure 2. (a)- Coordinate system and dimensions. (b)- Curvilinear transformation functions.

Coordinate transformation functions
$$X_1(\phi),\ X_3(\phi),\ Y_1(\phi),\ Y_3(\phi)$$

Curvilinear coordinate - ϕ

(a)

Coordinate transformation functions
$$X_5(\phi),\ Y_5(\phi)$$

Curvilinear coordinate - ϕ

(b)

Figure 3. Curvilinear transformation functions in circumferential direction

16

Coordinate lines r=const for 0°-ply with moulded-in hole   (a)

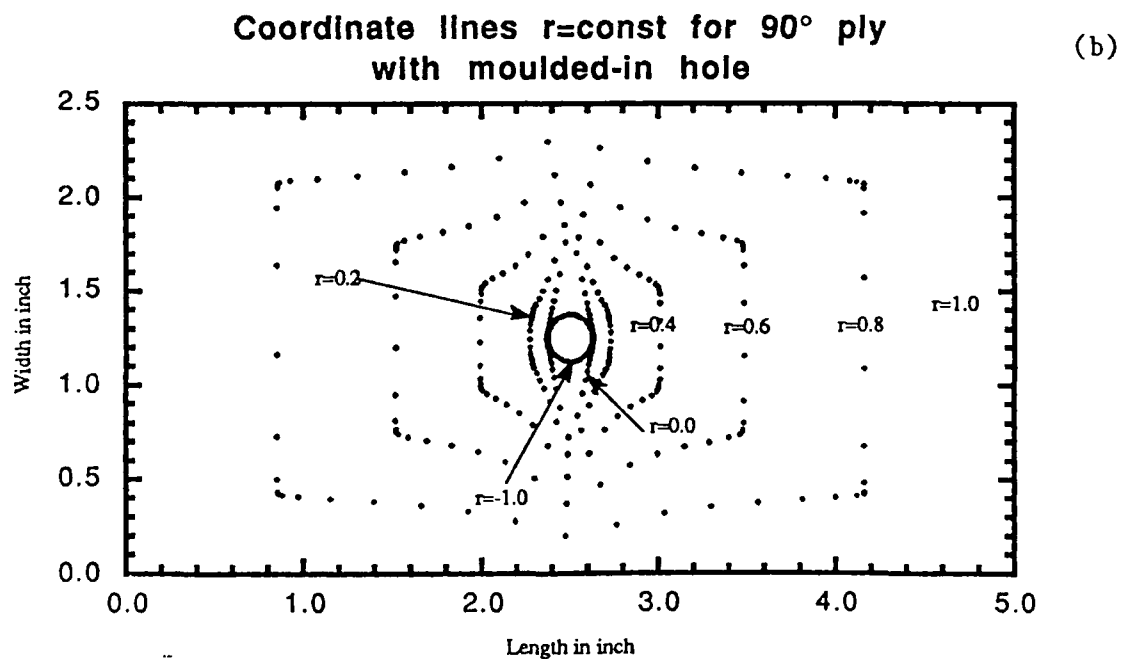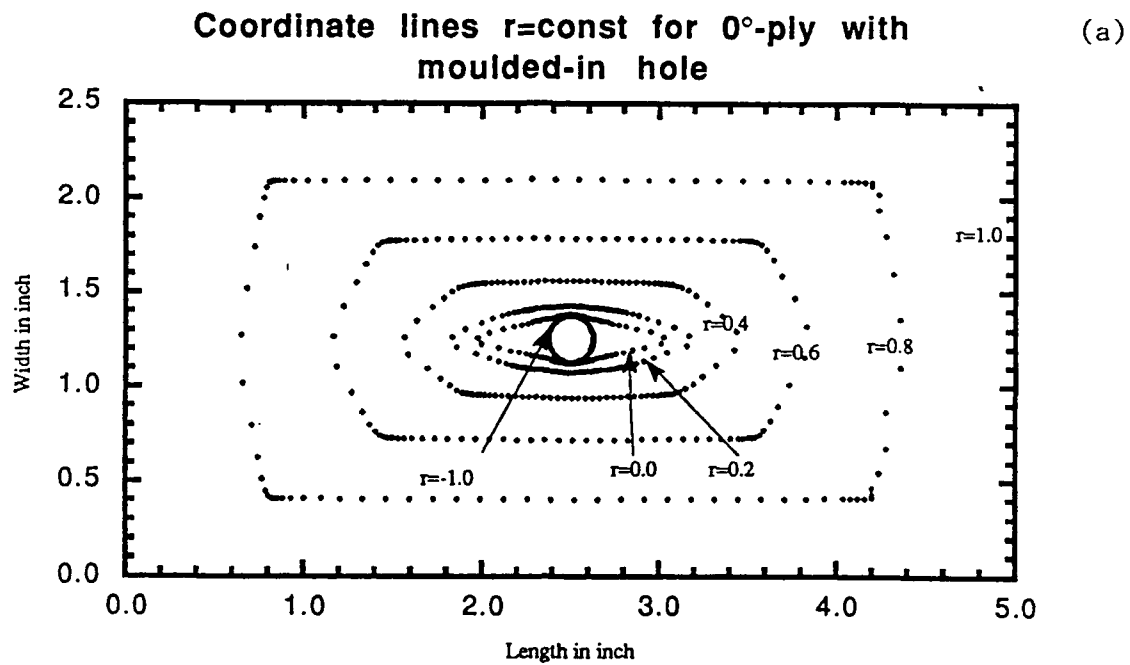**Coordinate lines r=const for 90° ply with moulded-in hole**   (b)

Figure 4.   The curvilinear transformation in the 0°-ply (a) and 90°-ply (b). The coordinate lines r = const.

Spline Functions $R_j(r)$ for
7 intervals between -1 and 0 and
7 intervals between 0 and 1

(a)

Spline functions in r direction graph with $R_j(r)$ on vertical axis (0.0 to 1.0) and r-coordinate on horizontal axis. Labels: $R_0 \cdots$, $R_9 \cdots$, R_{18}. Horizontal axis markers: $r_0 = -1$, r_2, r_4, r_6, 0, r_8, r_{10}, r_{14}, $r_{14} = 1$.

r - coordinate

Spline functions $\Phi_k(\phi)$ for
17 intervals between 0 and 2π

(b)

Spline functions in ϕ direction graph with $\Phi_k(\phi)$ on vertical axis (0.0 to 0.7). Horizontal axis markers: ϕ_{-3}, $\phi_0 = 0$, ϕ_4, ϕ_8, ϕ_{12}, $\phi_{17} = 2\pi$, ϕ_{20}.

additional nodes to provide the periodicity condition

ϕ - coordinate

additional nodes to provide the periodicity condition

Figure 5. Spline functions in r and ϕ directions : (a) and (b) respectively

18

Cubic spline functions $Z_i(z)$ for 7 sublayers through the thickness of the ply
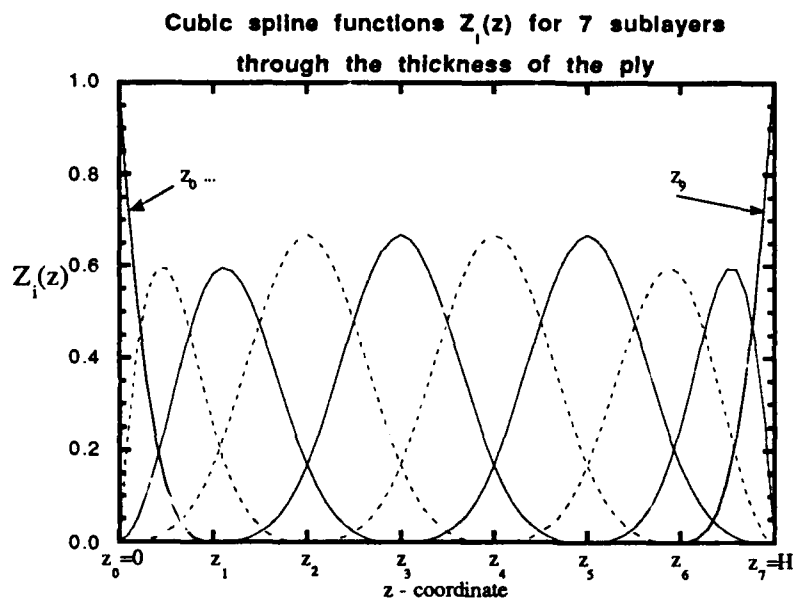
Figure 5.c. Spline functions in z-direction

Stresses in fiber direction in 0° ply

σ_{11}/σ_0

Gap filled with 90 fibers

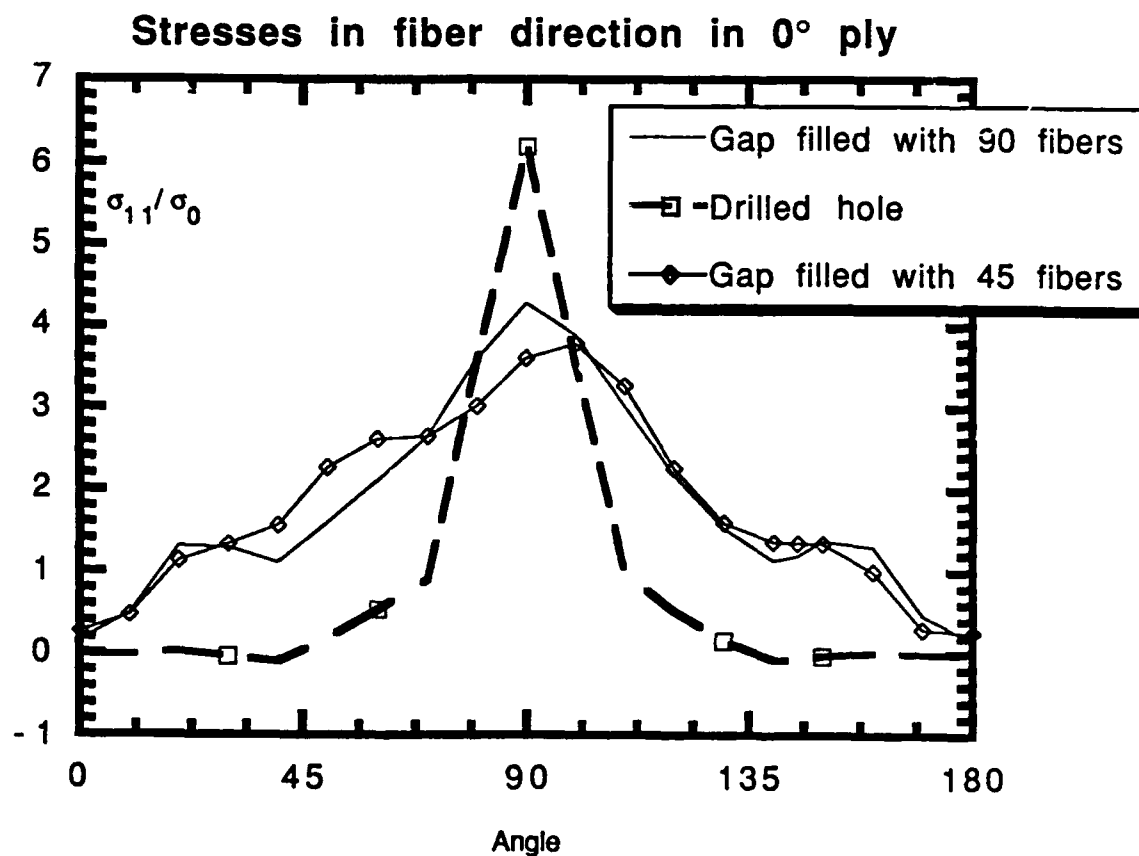Drilled hole

Gap filled with 45 fibers

Angle

Figure 6.   Stress in Region #1 for 0° ply at the gap edge with
molded hole and at the hole edge for drilled hole.

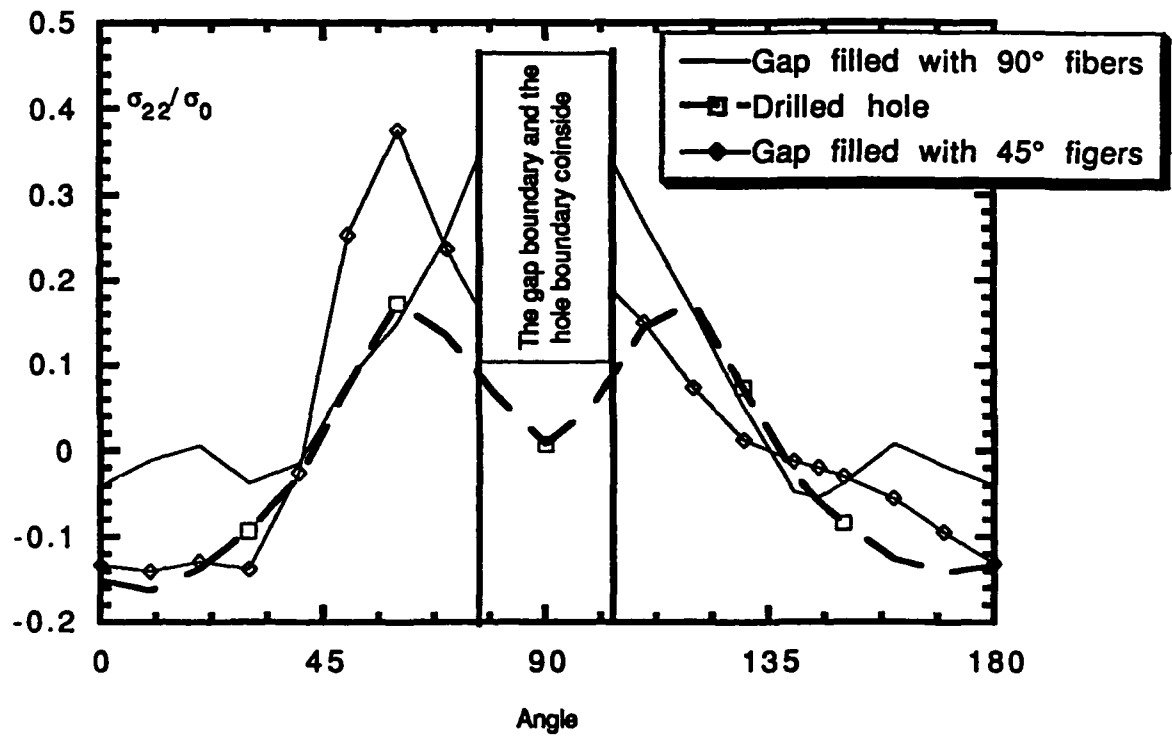**Stresses in the direction perpendicular to fibers in 0° ply**

Figure 7. Stress in the Region #2 of 0° ply at the hole edge with molded hole and at the hole edge for drilled hole.

21

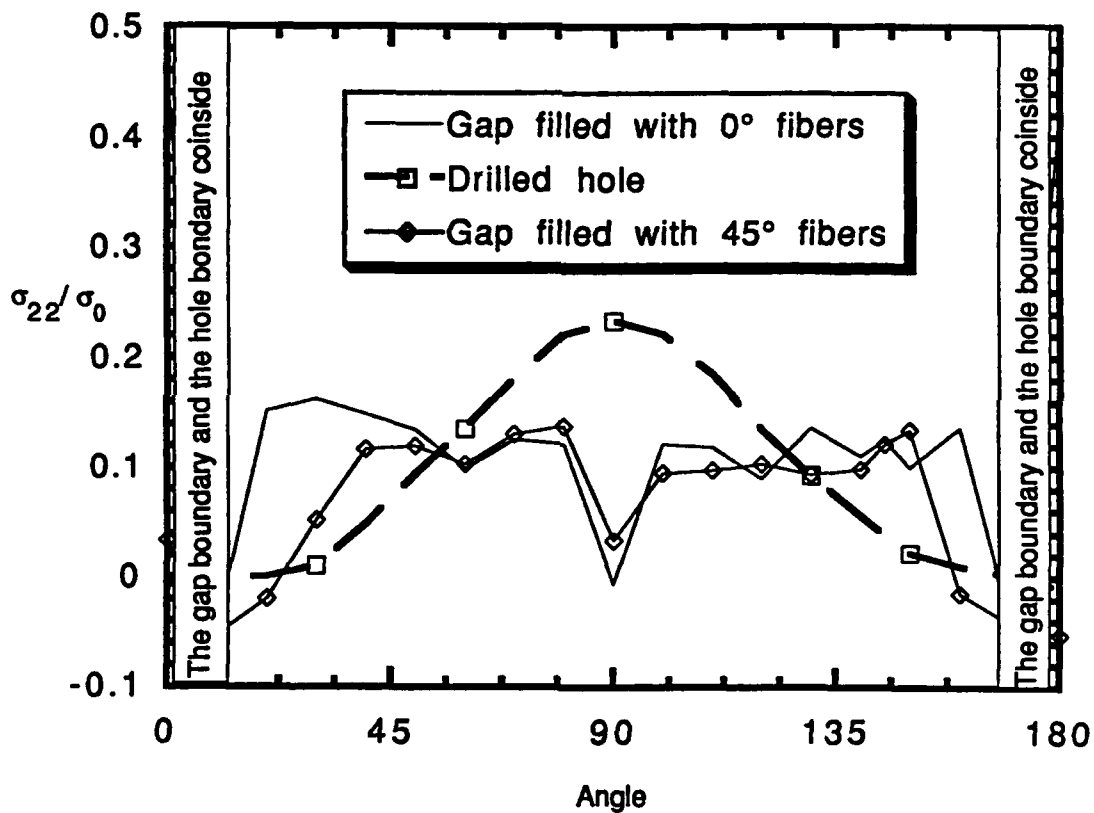**Stresses in the direction perpendicular to fibers in 90° ply**

Figure 8. Stress in Region #1 at the gap edge for 90° ply with molded hole and at the hole edge for drilled hole.
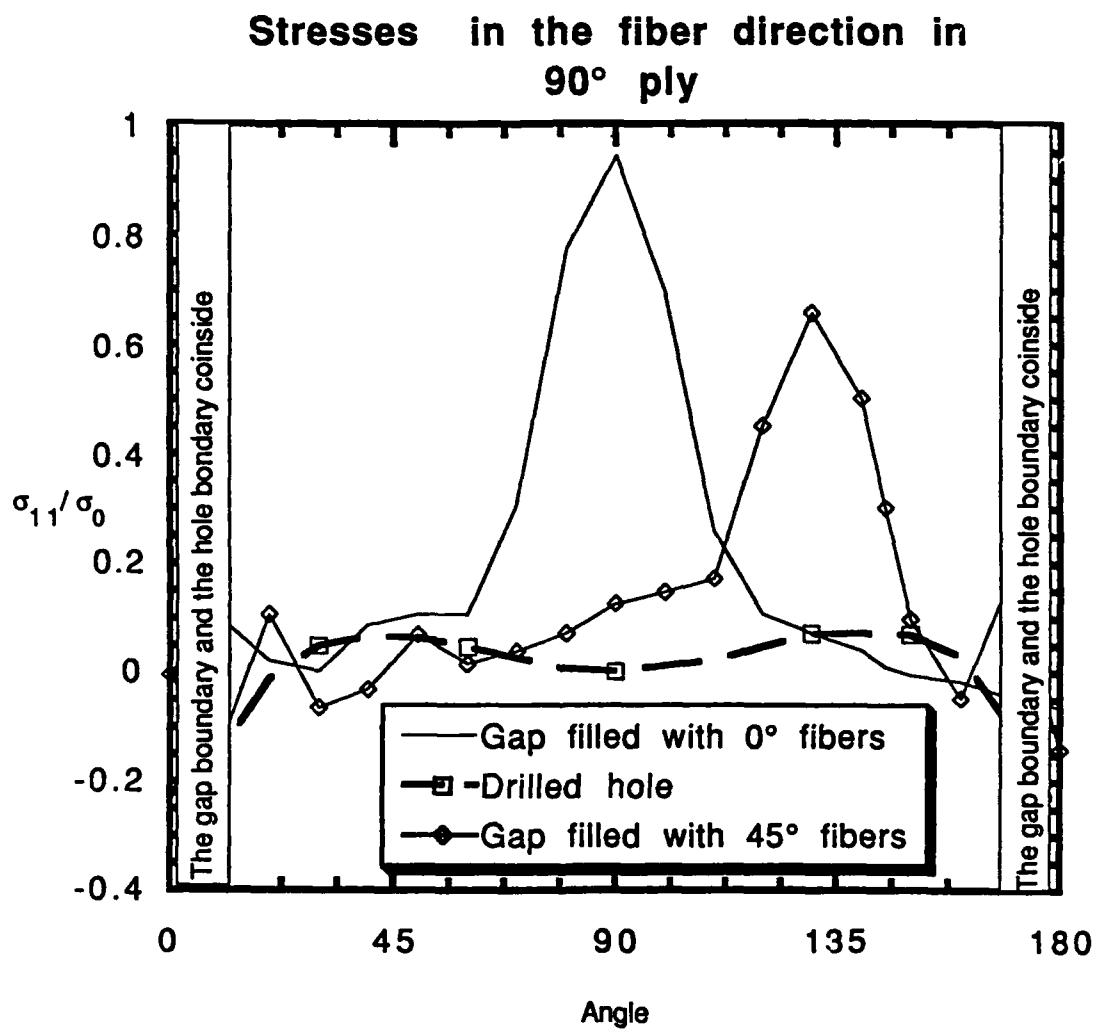
Figure 9. Stress in Region #2 at the hole edge for 90°
ply with molded and drilled hole.